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Breakdown of gases in a high-frequency electric field with and without a steady transverse magnetic field

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Abstract. Breakdown of gases at a pressure of 1.5 mtorr under the action of a highfrequency uniform electric field is studied in tubes of length 5 cm, 7 cm and 15 cm and 3.5 cm diameter, as well as with a superimposed steady transverse magnetic field of 18 gauss to 45 gauss in the tube of length 5 cm. It is found that the breakdown voltage increases as the length is decreased. The cut-off wavelength increases with the tube length. The breakdown is here due to secondary electron resonance. The phase angle ϕ and the maximum kinetic energy of the electron hitting the wall have been calculated and found to agree with earlier results. In the presence of a magnetic field the breakdown voltage, as well as the cut-off wavelength, increases with rising magnetic field. The corresponding theory is developed and the effect of the angle on arrival at the end wall on the breakdown mechanism is considered. Comparisons of the observations with the theory yield values of the phase angle and of the electron's kinetic energy on arrival at the wall with a fair amount of accuracy, especially for magnetic fields smaller than 30 gauss. The probability of electrons colliding with the side walls is considered. Under the present experimental set-up the majority of electrons with energy sufficient to cause breakdown travel freely between the two ends of the tube. The validity of some of the simplifying assumptions made in the theory is discussed.

1. Introduction

The mechanism of breakdown of a gas at pressures less than a few millitorr has been under investigation for some time to bring to light its salient features. Some of the workers, notably Hatch and Williams (1954, 1958), measured the starting peak voltage using internal electrodes and obtained a graph of starting peak potentials against frequency, which is a closed curve outside of which no discharge can be started. Experiments with external electrodes have not shown either an upper starting potential or the extinction of the discharge by an increasing field, except at extremely low pressure (Francis 1960). Early investigations of Gutton (1924), Gutton and Gutton (1928), Gutton (1930) and Kirchner (1925, 1930) and later by Backmark and Bengston (1941) led to a theoretical analysis of the mechanism by Danielsson (1943). This type of discharge has been attributed to the emission of secondary electrons from the opposite walls of the discharge tube, and a detailed theory has been worked out by Gill and von Engel (1948), which predicts a cut-off law relating cut-off frequency and electrode separation. The phenomena has also been explained on the assumption of the process of electron bunching in multipacting discharge, otherwise known as discharge initiated by the motion of secondary electrons in resonance with an external alternating electric field at pressures less than 10^{-3} torr, by Miller and Williams (1962), Paschke (1961) and Hatch (1961). Though a consistent theory of the phenomena has been developed by Gill and von Engel (1948), it is worth while to investigate some of the consequences of the theory with regard to variation of the starting voltage and the cut-off frequency with the length of the discharge tube.

The effect of superimposing an external electric field upon this type of discharge was investigated by Kossel and Krebs (1954), although no quantitative explanation of the observed results was provided. It has been found that superimposing a d.c. electric field parallel to a high-frequency electric field can make starting more difficult. A small static magnetic field perpendicular to the high-frequency electric field causes a general increase of breakdown potential and a lowering of the cut-off frequency without changing the nature of the (E, λ) curve, where E is the starting peak potential and λ is the wavelength of the applied radio-frequency field. At large magnetic fields the starting potential becomes independent of frequency, and for very low pressure (10^{-5} torr) the discharge can be extinguished either by increasing the electric field or decreasing the magnetic field. Deb and Goswami (1964) made a theoretical approach to the problem when a steady magnetic field is established perpendicular to a high-frequency electric field. No systematic observation of the breakdown of gases controlled by secondary electron emission in a high-frequency electric field in the presence of an external d.c. magnetic field has so far been undertaken. The object of the present investigation is thus to study the effect of a transverse d.c. magnetic field on this type of breakdown with regard to the starting field and the cut-off frequency. The theory of the previous workers has to be modified owing to the effects produced by the introduction of the magnetic field, and it is presumed that these investigations may throw some light on the mechanism of such discharges.

2. Experimental arrangement

The breakdown potential of the gas has been determined as by Gill and von Engel (1948). The radio-frequency electric field is a tuned-plate tuned-grid oscillator covering the frequency range of 4 to 30 MHz in three stages. The output of the oscillator, which can be varied from 0 to 500 v, has been measured by a vacuum-tube voltmeter; measurements of the breakdown voltage at the highest-frequency end is limited by the radio-frequency voltage output of the oscillator. The discharge tubes of 3.5 cm diameter, made of Pyrex glass, are properly cleaned and evacuated to 10^{-5} torr by an oil diffusion pump. The plane circular electrodes, which are external and perpendicular to the tube axis, are connected to the radio-frequency source (figure 1). Three discharge tubes of length 5 cm, 7 cm and



Figure 1. Experimental set-up: all the dimensions are given in cm.

15 cm were used to study the effect of the length of the discharge tube on the breakdown potential as well as on the cut-off frequency. Besides pure dry air, hydrogen, prepared by the electrolysis of barium hydroxide solution and dried by phosphorous pentoxide, has been used. No special attempt for the purification of the gases has been made as neither the gas nor the impurity have any effect upon this type of discharge. All the measurements have been made at a pressure of 1.5 mtorr, which has been measured by an Edward–Penning– Pirani vacuum gauge. The steady magnetic field has been provided by an electromagnet having flat pole pieces of area 7.5 cm $\times 4.5$ cm, placed at right angles to the tube axis. The experiment with a transverse steady magnetic field has been performed with the tube of 5 cm length only, so that the tube remains well inside the magnetic pole pieces to ensure a uniform magnetic field, which has been measured by a calibrated fluxmeter. Except for the external electrodes, the system has been properly grounded. As the voltage is gradually increased a faint glow appears at the breakdown point, and simultaneously there is a slight drop in the output voltage at the vacuum-tube voltmeter. This drop in voltage is less and less marked as the cut-off frequency is approached.

3. Results and discussion

3.1. Low-pressure breakdown in a uniform high-frequency electric field

To start with, starting potentials have been measured in air and hydrogen in a discharge tube of length 15 cm to verify the argument that breakdown controlled by secondary electron emission is independent of the nature of the gas. The full curve in figure 2



Figure 2.

represents the results in the case of air and the circles on it are the observations with hydrogen. The identical nature of the two breakdown curves indicates that the type of breakdown observed in the present experimental set-up is independent of the nature of the gas. Starting potentials have been measured in three discharge tubes of length 5 cm, 7 cm and 15 cm (each tube 3.5 cm in diameter), and the results have been plotted in figure 3.



Figure 3.

It is observed that the starting potential is higher in tubes of shorter lengths and the cut-off wavelength increases with the length of the discharge tube. Measurements towards the shorter-wavelength region could not be taken owing to a limitation in the output power of the oscillator. It is evident that the curves are identical with those obtained by previous workers, showing a sharp cut-off towards higher wavelengths.

The qualitative description of the mechanism, viz. the oscillation of electrons at resonance with the applied high-frequency electric field and their multiplication by the release of secondary electrons from the end walls, has been presented by Danielsson (1943), Gill and von Engel (1948), Hatch and Williams (1954, 1958) in almost identical ways.

It has been shown by Gill and von Engel (1948) that the starting field E is given by

$$E = \frac{p^2 d}{(e/m)\Phi} \tag{1}$$

where

$$\Phi = \frac{K+1}{K-1}\pi\cos\phi + 2\sin\phi \tag{2}$$

where d is the distance between the end walls, $K = v/v_0$ and P is the angular frequency of the applied high-frequency field. Gill and von Engel analysed their data using these equations and taking ϕ as a parameter. Hatch and Williams (1954) have shown that, if the electron arrival energy is \mathscr{E} ev, then we obtain for the frequency

$$f = \frac{(K-1)\Phi}{K\pi d\cos\phi} \left(\frac{\mathscr{E}}{8m}\right)^{1/2}.$$
(3)

Following Hatch and Williams, let us assume K = 3 and equations (1) and (2) are fitted to the experimental curve (figure 3). Some points are chosen in each curve and the calculated values of corresponding ϕ are marked there. The values of ϕ so obtained in all the three tube lengths of 5 cm, 7 cm and 15 cm are not much different from the values obtained by previous workers. For constant K, ϕ_{co} and \mathscr{E}_{crit} , we obtain from equation (3)

$$f_{\rm co}d = \rm constant \tag{4}$$

where the constant of equation (4) is obtained by fitting the experimental curve. Gill and von Engel (1948) obtained the value of the constant from a general similarity theorem to be 79. This value increases with the increase of the length of the tube, and it has been attributed to a side wall effect and distortion of the uniformity of applied field at large separation, and has been explained clearly by Gill and von Engel (1948), Hatch and Williams (1954, 1958) and recently by Chandrakar and von Engel (1965). We fitted the above equations to our experimental curves, and the different quantities as calculated are shown in table 1.

Table 1

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
15	3.5	8	120	9.3	$-52\cdot3^{\circ}$	102.5	1.36
5	3.5	24	119	24	-44.3	/3.5	1.30
5	5.5	24	120	27		00	1.20

(1) Length d of discharge tube (cm); (2) diameter of discharge tube (cm); (3) frequency f_{co} at cut-off (MHz); (4) $f_{co}d$; (5) starting voltage E_{co} at cut-off point (v cm⁻¹); (6) phase angle ϕ_{co} at cut-off (deg); (7) energy \mathscr{E}_{crit} of electron at cut-off (ev); (8) C (v ev⁻¹).

From the table it can be seen that the values of ϕ_{co} and \mathscr{E}_{crit} are not much different from the values obtained by previous workers and the experimental value of the energy of 80 ev for $\delta = 1$, where δ is the secondary emission coefficient. In figure 4 the cut-off law given by equation (4) is represented for the value of the constant 79 (Gill and von Engel 1948) and also for 120, the value obtained in the present work. The results of Gutton (1924), Gutton (1930), Gill and von Engel (1948), Hatch and Williams (1954, 1958) and those of the present work are plotted in figure 4 for comparison. It is found that for a tube length larger than 2 cm equation (4) fits the experimental results better when the constant



Figure 4. △ Gutton 1924, Gutton 1930; □ Gill and von Engel 1948; ○ Hatch and Williams 1954; ● present work.

is taken as 120. If we assume ϕ_{co} and K to be constant at cut-off, the breakdown voltage at cut-off is

$$V_{\rm co} = C \mathscr{E}_{\rm crit} \tag{5}$$

where C is a constant; equation (5) is independent of wall separation and applied frequency. The value of the constant C is obtained by fitting equation (5) to the present work and the results are entered in the last column of table 1. It is seen that the value of C is a constant as predicted by relation (5), and when $\mathscr{E}_{\text{crit}}$ is expressed in ev and V_{co} in volts the value of C is 1.36.

It can thus be stated that the theory of Gill and von Engel fits in with a fair amount of success with our different observations. The values obtained of different unknown parameters involved in the theory for the process of fitting show a fair amount of consistency among themselves for different lengths of the tube and when compared with the values obtained by previous workers.

3.2. Low-pressure breakdown in a uniform high-frequency electric field with a uniform transverse d.c. magnetic field

Here the equation of motion of an electron is

$$\frac{\mathrm{d}\nu}{\mathrm{d}t} = -\frac{e}{m}(E + \nu \times H). \tag{6}$$

As its solution in simple form is unattainable, following Deb and Goswami (1964), it is assumed that $\alpha \pi = 2n\pi$ (*n* is an integer), where $\alpha = \omega/p$ and $\omega = eH/m$, the cyclotron frequency. The resultant velocity is given by

$$v^{2}\left(1-\frac{1}{K^{2}}\right) = 4\left[-\frac{vpeE\cos\phi}{Km(\omega^{2}-p^{2})} + \left\{\frac{eEp\cos\phi}{m(\omega^{2}-p^{2})}\right\}^{2} + \left\{\frac{eE\omega\sin\phi}{m(\omega^{2}-p^{2})}\right\}^{2}\right]$$
(7*a*)

and the displacements at $t = \pi/p$ are

$$x = -\frac{2eE\sin\phi}{m(\omega^2 - p^2)} \tag{7b}$$

$$y = \frac{2eE\omega\cos\phi}{mp(\omega^2 - p^2)}.$$
(7c)

If δ denotes the coefficient of maximum secondary electron yield, then from Bruining (1954), for a metallic surface,

$$\frac{\delta_{\theta}}{\delta_0} = \exp\{C'(1 - \cos\theta)\}\tag{8}$$

where C' is a constant which is different for different surfaces. If relation (8) is assumed to be true for glass, then C' becomes almost equal to unity (Deb and Goswami 1964). Hence, for glass,

$$\frac{\delta_{\theta}}{\delta_0} = \exp\{1 - \cos\theta\} \tag{9}$$

where δ_{θ} is the value of δ for an angle of incidence θ of the primary electron and δ_0 is that for normal incidence.

Mueller's (1945) results on hard glass are given in figure 5, which shows the average number of electrons released from the glass surface, irrespective of the angle of emission,



Figure 5.

as a function of the speed of the primaries. From this curve an empirical relation between \mathscr{E} , the energy of the primary electrons in ev, and δ , the yield, can be represented by the equation

$$\delta = 1 \cdot 2 \left(\frac{\mathscr{E}}{100}\right)^{2/3}.$$
 (10)

This relation is fitted to the experimental curve, which shows that the fitting is valid between 80 ev and 300 ev of the energy of the primary. In secondary resonance breakdown in a Pyrex glass vessel the primary energies lie within this limit, and hence this relation between δ and \mathscr{E} can be utilized. Combining relations (9) and (10), we obtain

$$\mathscr{E}_{\text{eff}} = \mathscr{E} \exp\{\frac{3}{2}(1 - \cos\theta)\}$$
(11)

where \mathscr{E}_{eff} and \mathscr{E} are the primary electron energies for hitting the surface at angles θ° and 0° , respectively, for the same maximum secondary yield. Substitution of equation (11) in the general solution of equation (1) shows that the effect of oblique hitting of the primary can well be accounted for by replacing E by $E \exp\{\frac{3}{4}(1-\cos\theta)\}$ in any solution of equation (1). The y displacement at $t = \pi/p$ for different values of magnetic field are calculated from equation (7c) and are given in the last column of table 2. The resultant displacement $(x^2 + y^2)^{1/2}$ at $t = \pi/p$ for minimum and maximum values of the magnetic field lies between 5.297 and 5.702, and the length of the tube is 5 cm. Consequently, as a first approximation, the resultant displacement can be taken to be equal to the length of the tube. Hence

$$\frac{dm(\omega^2 - p^2)p}{2eE\Phi_{\rm H}} = \exp\{\frac{3}{4}(1 - \cos\theta)\}$$
(12)

where

$$\Phi_{\rm H} = (\omega^2 \cos^2 \phi + p^2 \sin^2 \phi)^{1/2}.$$

As $\frac{3}{4}(1-\cos\theta) < 1$ and $\tan\theta = -\alpha \cot\phi$, equation (12) reduces to a quadratic equation in $\sin\phi$: $\sin^2\phi(49\omega^2 - 40p^2) + 24\Lambda_{\rm H}p\sin\phi + (16\Lambda_{\rm H}^2 - 49\omega^2) = 0$ (13) where

$$\Lambda_{\rm H} = \frac{md(\omega^2 - p^2)p}{2eE}$$

For different values of $\Lambda_{\rm H}$, ω and ϕ equation (13) is solved and values of the phase angle ϕ are chosen, depending upon the portion of the curve under consideration. In figure 6



Figure 6. (1) H = 0 gauss; (2) H = 18 gauss; (3) H = 21 gauss; (4) H = 30 gauss; (5) H = 45 gauss.

experimental curves for magnetic field values of 18 gauss, 21 gauss, 30 gauss and 45 gauss are given up to the highest range of frequency at which measurements are limited by the radio-frequency voltage output of the present experimental set-up. Values of ϕ obtained from equation (13) for some points on these curves are marked and also given in table 2.

Table 2

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
18 21 30	22 21 18·5	38 44 54 76	62 52 58	4.3 3.57 3.1 1.84	52 35·85 27·04	88.4 86.04 73.35	88	1.327 0.9445 0.5741	1.75 2.228 2.601

(1) H (gauss); (2) f_{co} (MHz); (3) E_{co} (v cm⁻¹); (4) $-\phi_{co}$ (deg); (5) $v_{co} \times 10^{-8}$ (cm s⁻¹); (6) \mathscr{E}_{orig} (ev); (7) \mathscr{E}_{eff} (ev); (8) \mathscr{E}_{orit} from table 1 (ev); (9) Larmor radius (cm); (10) y displacement from equation (7c) (cm).

We have confined our discussion and fittings to the almost linear portion of the curve up to the cut-off point, which could be obtained in the present experimental set-up, where the energy of an electron is almost constant (Hatch and Williams 1954). Knowing ϕ_{co} from equation (13) and solving equation (7*a*), we have calculated the kinetic energy of arrival \mathscr{E} , and, using the relation (11), the values of \mathscr{E}_{eff} for the cut-off points of each value of the magnetic field have been obtained and are shown in table 2.

From table 2 it is seen that the value of $\mathscr{E}_{\text{orig}}$ is much smaller than the critical value of $\mathscr{E}_{\text{orit}}$ calculated from the cut-off point of the curve at H = 0. But the value of \mathscr{E}_{eff} for H up to 30 gauss is near or equal to $\mathscr{E}_{\text{orit}}$ for H = 0, although the value shows a gradual decrease as H is increased. The remarkable deviation of \mathscr{E}_{eff} from $\mathscr{E}_{\text{orit}}$ for H = 45 gauss may be due to different approximations made in calculating ϕ and v from a comparison with the experimental results. The different terms neglected modify the values of the parameters to a large extent for a high magnetic field. However, if we consider the limitations of our theory in explaining the mechanism of the discharge and the experimental results by the process of fitting, it can be said that the agreement is fairly good, at least for moderate values of the magnetic field. This treatment also shows that the mechanism of secondary electron resonance is still operative in the original sense as the cause of the breakdown of the gas, when the magnetic field is also present. It is also expected that, if the original solutions of the equation of motion could be obtained, much better fitting of the experimental results with more reasonable values of the parameters could be achieved.

The values of the Larmor radius and those of y at $t = \pi/p$ have been calculated for different values of the magnetic field and given in columns (9) and (10) of table 2, respectively. The y displacement when the electrons reach the opposite end for each of the values of the magnetic field is smaller than the diameter of the discharge tube (3.5 cm); the Larmor radius in each case is much smaller than the radius of the tube for energies of electrons high enough to cause breakdown, and the majority of electrons which are actually responsible for the continuance of the secondary electron resonance breakdown find ample free space during their transit between the end walls and are not lost owing to collision with the side walls.

It is further observed from table 2 that an empirical relation between H, f_{co} and E_{co} can be obtained from the experimental data. The quantity Hf_{co}/E_{co} where H is expressed in gauss, f_{co} in MHz and E_{co} in v cm⁻¹ is almost a constant, as shown in table 3.

Table 3

H (gauss)	$f_{ m co}$ (MHz)	$E_{\rm co}$ (V cm ⁻¹)	$rac{Hf_{ m co}}{E_{ m co}}$	$-\phi_{\circ\circ}$ (deg)	$ heta_{\circ\circ}$ (deg)	$\frac{Hf_{co}}{E_{co}}$ (calc.)
18	22	38	10.42	62	51	5.088
21	21	44	10.02	52	66	6.7
30	18.5	54	10.27	58	77	6.119
45	16.5	76	9.77	34	85	10.1

To test whether the theoretical analysis made above can explain the empirical relation observed, we obtain from equation (12), with $\phi = \phi_{co}$, $p^2/\omega^2 \ll 1$ for the magnetic field used in this experiment and using $\tan \theta_{co} = -\alpha \cot \phi_{co}$,

$$\frac{f_{\rm co}H}{E_{\rm co}} = 6.37 \frac{\cos\phi_{\rm co}}{\sin\theta_{\rm co}} \exp\{\frac{3}{4}(1 - \cos\theta_{\rm co})\}$$
(14)

where f_{co} is in MHz, H in gauss and E_{co} in v cm⁻¹. Experimentally we find that

$$\frac{Hf_{\rm co}}{E_{\rm co}} \simeq 10$$

so that we may say that θ_{co} and ϕ_{co} adjust themselves in such a way that at the point of cut-off given by equation (14) the value of the right-hand side of the equation remains constant for any magnetic field. To test the validity of this conclusion, the individual values of the right-hand side of equation (14) are calculated for each value of the magnetic field. The results in the last column of table 3 are of the same order of magnitude as the experimentally determined value of the constant. The discrepancy may be attributed to the different approximations taken and their validity during the theoretical deductions.

4. Conclusion

The phenomenon of low-pressure breakdown by secondary electron resonance oscillation has been explained in the light of the theory put forward by Gill and von Engel and Hatch and Williams. The measurements without magnetic field lead to the conclusion that all the predictions of the theory of Gill and von Engel (1948) can be extended for a wide range of the dimensions of the discharge tube. The increase in the value of the constant $f_{co}d$ at cut-off, compared with the previous work with a small gap length of discharge tube, justifies to some extent the predicted reasoning of Hatch and Williams (1954), and supported recently by the work of Chandrakar and von Engel (1965), that this is due to side wall effects, especially when the length of the discharge tube is large. The quantity $V_{\rm co}/\mathscr{E}_{\rm crit}$ has been found to be a constant for all the three lengths of the discharge tube—a result which follows from theory.

The observations with magnetic field and the subsequent fitting of these observations to our theory yielded values of the phase angle which are reasonable. Though our procedure has obtained a simplified form for the energy of arrival of an electron, yet it gives results for the effective energy of arrival for a moderate magnetic field with a fair amount of accuracy.

The validity of the different assumptions made for the deduction of the simplified theory of breakdown with a magnetic field are open to questions in the rigorous sense. Moreover, in view of the fact that no theory of breakdown in a magnetic field can be developed without the use of simplifying assumptions and in view of the usefulness of this theory in explaining the experimental results, it can be concluded that the assumptions made can be regarded as valid in the range of the magnetic field studied in the present investigations.

References

BACKMARK, N. E., and BENGSTON, U., 1941, Diplomarbeit, K. Tech. Hochschule Stockholm.

- BRUINING, H., 1954, Physics and Applications of Secondary Electron Emission (Oxford: Pergamon Press).
- CHANDRAKAR, K., and VON ENGEL, A., 1965, Proc. R. Soc. A, 284, 442.

DANIELSSON, U., 1943, Diplomarbeit, K. Tech. Hochschule Stockholm.

DEB, S., and GOSWAMI, S. N., 1964, Br. J. Appl. Phys., 15, 1501.

FRANCIS, G., 1960, Ionization Phenomena in Gases (London: Butterworths).

GILL, E. W. B., and VON ENGEL, A., 1948, Proc. R. Soc. A, 192, 446.

GUTTON, C., 1924, C.R. Acad. Sci., Paris, 178, 467.

GUTTON, C., and GUTTON, H., 1928, C.R. Acad. Sci., Paris, 186, 303.

GUTTON, H., 1930, Ann. Phys., Lpz., 13, 62.

HATCH, A. J., 1961, J. Appl. Phys., 32, 1086.

HATCH, A. J., and WILLIAMS, H. B., 1954, J. Appl. Phys., 25, 417.

----- 1958, Phys. Rev., 112, 681.

KIRCHNER, F., 1925, Ann. Phys., Lpz., 77, 287.

----- 1930, Ann. Phys., Lpz., 7, 798.

KOSSEL, F., and KREBS, K., 1954, Z. Phys., 139, 189.

MILLER, A., and WILLIAMS, H. B., 1962, J. Appl. Phys., 33, 1621.

MUELLER, E. W., 1945, J. Appl. Phys., 16, 453.

PASCHKE, F., 1961, J. Appl. Phys., 32, 747.

PASCHKE, F., and KREBS, K., 1954, Z. Phys., 139, 189.

WILLIAMS, H. B., and HATCH, A. J., 1953, Phys. Rev., 89, 339.